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p.g. sem-2 28-04-2k2 paper-VI unit complex integration Dr. G.D. Singh.

Cauchy's Integral Formula:

Theorem:→ If $f(z)$ is analytic within and on a closed contour C, and if a is any pt within C, then

$$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$$

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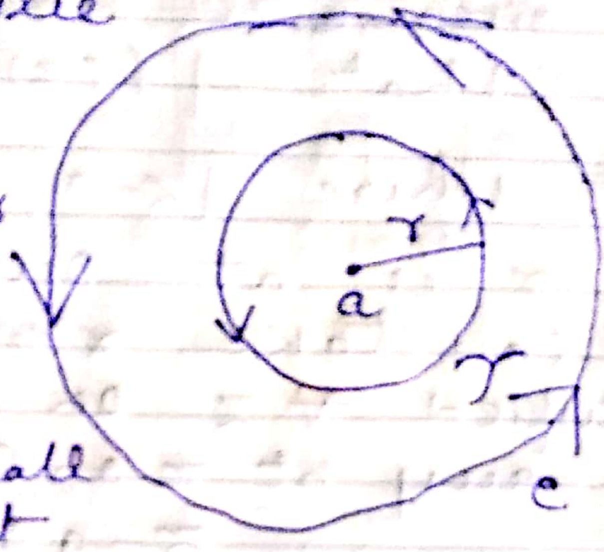
Let $f(z)$ be analytic within and on a closed contour C and a is an interior point of C.



We describe a circle Γ about the centre $z = a$ of small radius r such that this circle $|z-a| = r$ does not intersect the curve (contour) C.

The function $\dfrac{f(z)}{z-a}$ is analytic in the annulus bounded by C and Γ.

So, by Corollary to Cauchy's theorem

$$\int_C \frac{f(z)}{z-a}\,dz = \int_\Gamma \frac{f(z)}{z-a}\,dz \quad \text{——— (1)}$$

i.e.
$$\int_C \frac{f(z)}{z-a}\,dz = \int_\Gamma \frac{f(z)-f(a)+f(a)}{z-a}\,dz$$

$$= \int_\Gamma \frac{f(z)-f(a)}{z-a}\,dz + \int_\Gamma \frac{f(a)}{z-a}\,dz \quad - (2)$$

Since $f(z)$ is analytic within C and so it is continuous at $z = a$ so that

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given $\epsilon > 0$ there exists $\delta > 0$ such that $|f(z) - f(a)| < \epsilon$ —— (3.)

where $|z-a| < \delta$ —— (4.)

As r is at our choice and so we can take $r < \delta$ and so (4) is satisfied $\forall z$ on the circle Γ.

For any pt z on Γ,

$$z - a = re^{i\theta}$$

$$\therefore dz = re^{i\theta} i\,d\theta$$

$$\therefore \int_{\Gamma} \frac{f(a)}{z-a}\,dz = \int_0^{2\pi} \frac{f(a)\,re^{i\theta}\,i\,d\theta}{re^{i\theta}}$$

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$$= i f(a) \int_0^{2\pi} d\theta$$

$$= i f(a) \left[\theta\right]_0^{2\pi}$$

$$= i f(a) \left[2\pi - 0\right] = 2\pi i f(a)$$

Thus by (2)

$$\int_C \frac{f(z)}{z-a} = 2\pi i f(a) + \int_{\Gamma} \frac{f(z) - f(a)}{z-a}\,dz$$

Or, $$\int_C \frac{f(z)}{z-a}\,dz - 2\pi i f(a) = \int_{\Gamma} \frac{f(z)-f(a)}{z-a}\,dz$$

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or, $\left|\int_C \frac{f(z)\,dz}{z-a} - 2\pi i f(a)\right| = \left|\int_\Gamma \frac{f(z)-f(a)}{z-a}\,dz\right|$

or $\left|\int_C \frac{f(z)\,dz}{z-a} - 2\pi i f(a)\right| \leq \int_\Gamma \frac{|f(z)-f(a)|}{|z-a|}\,|dz|$

$$< \frac{\epsilon}{r}\int_\Gamma |dz|$$

$$< \frac{\epsilon}{r}\cdot 2\pi r$$

$$< 2\pi\epsilon$$

Since ϵ is arbitrary and making $\epsilon \to 0$ we get.

$$\int_C \left\{\frac{f(z)}{z-a}\,dz - 2\pi i f(a)\right\} = 0$$

or $\int_C \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$

Or $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$ Proved

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$\because \int_\Gamma |dz| = \int_0^{2\pi r} dz = [z]_0^{2\pi r} = 2\pi r.$